

Gradient-Index Ophthalmic Lens Design and Polymer Material Studies

by
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Curriculum Vitae

The author was born in Indianapolis, Indiana on September 8, 1971. He attended Rose-Hulman Institute of Technology from 1989 to 1993. In May 1993, he graduated Summa Cum Laude with a Bachelor of Science in Applied Optics. In the fall of 1993, he entered the doctoral program at The Institute of Optics at the University of Rochester. From the summer of 1997 to late winter in 1998, he worked as Optical Engineer at Chapman Instruments in Rochester, NY. His research in gradient-index optics was supervised by Professor Duncan T. Moore.

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Abstract

Unifocal ophthalmic lenses are conventionally designed using homogeneous glass or plastic materials and aspheric surfaces. The desired power and aberration correction are provided by selection of surface shape and refractive index. This thesis studies the design of ophthalmic lenses utilizing gradient-index (GRIN) materials for both the optical power and aberration control. This is done using geometrical optical theory and ray-tracing simulations.

Progressive addition lenses (PALs) are vision correction lenses with a continuous change in power used to treat presbyopia. The power variation is typically located in the lower half of the lens. Progressive addition lenses are currently made with aspheric surfaces to achieve the focal power transition and aberration control. These surfaces have at most, mirror symmetry about the vertical axis. The possible design of progressive addition lenses with GRIN materials has not been well studied. This thesis studies PALs and identifies how gradient-index materials can be used to provide both the power progression and aberration control. The optical theory for rotationally symmetric and asymmetric power additions is given. Analytical and numerical methods for calculating the index profile are used, and the results examined using ray-tracing simulations.

The theory developed for ophthalmic lenses is applied to the design of GRIN axicon. This is the first GRIN axicon manufactured, and is fabricated using ion-exchanged GRIN glass. Experimental measurements of its performance are compared and found to match theoretical predictions. This demonstrates the generality of the

theory developed: it may be applied to non-visual applications, and even to non-imaging applications.

Realistic implementation of GRIN technology to ophthalmic application requires the fabrication of large scale refractive index gradients in polymer material systems. The methyl-methacrylate/styrene copolymer system is studied to develop an empirical model of its time-dependent diffusion process. The diffusion of styrene into partially polymerized methyl-methacrylate is found to be Fickian, with a concentration-independent diffusion coefficient.

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